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# Modelling Social Network Sites with PageRank and Social Competences

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**Abstract.** In this communication a recent method to classify the users of an SNS into *Competitivity groups* is recalled. This method is based on the PageRank algorithm. *Competitivity groups* are sets of nodes that compete among each other to gain PageRank via the *personalization vector*. Specific features of the SNSs (such as number of friends or activity of the users) can be considered as *Social Competences*. By means of these *Social Competences* a node can modify its ranking inside a *Competitivity group*.

*Keywords:* Google matrix, PageRank, sns, ties, link analysis, ranking algorithm, online relationships
*MSC 2000:* 94C15, 15A51, 68P20

## 1. Introduction

Social Network Sites are here to stay since human beings are social beings. We need relationships, and accessing online relationships is our natural way to connect in the era of Web 2.0. SNSs have humanized the Internet.

*Computational Social Science* [1], [2] has emerged from the new foundations of the theory of Complex Networks [3], [4], [5].

In the area of Social Networks, links (*ties*) between people have been studied extensively by the sociologists [6], but using small, local and static samples. Now the definition of a link is under analysis: what is a link? what are the links that matter?

In this paper we are interested in classify users of SNSs. Using some new definitions [7] based on PageRank [8]. We propose to use the personalization vector as a measure of the social skills of the users of an SNS.

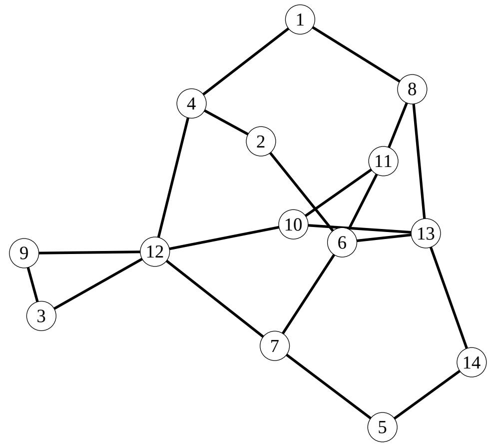


Figure 1: Network ScaleFree-4-2-10

## 2. Definitions

Given a directed graph, let $\mathcal{N}$ be the set of $n$ nodes, and $0 < \epsilon < 1$. Let $\pi$ be the Perron vector of the Google matrix associated to the graph. We call basic PR to the Perron vector obtained when using the usual personalization vector, i.e., $\mathbf{v} = \mathbf{e}/n$, with $\mathbf{e}$ the vector of all ones. Some new definitions introduced in [7] are the following.

**Definition 1** *Let* $\mathbf{v}_i = [v_{ij}] \in \mathbb{R}^{n\times 1} : v_{ii} = 1 - \epsilon$, $v_{ij} = \epsilon/(n-1)$ *if* $i \neq j$. *For each* $i \in \mathcal{N}$, *let* $PR_i = \pi(\mathbf{v}_i)$ *and we denote as* $(PR_i)_j$ *the* $j$*-th entry of* $PR_i$.

**Definition 2** *A* Competitivity group *is a subset of* $\mathcal{N}$. *Nodes* $i \in \mathcal{N}$ *and* $j \in \mathcal{N}$ *belong to the same* Competitivity group *if*

$$[\min_{k\in\mathcal{N}} (PR_k)_i, \max_{k\in\mathcal{N}} (PR_k)_i] \cap [\min_{k\in\mathcal{N}} (PR_k)_j, \max_{k\in\mathcal{N}} (PR_k)_j] \neq \emptyset.$$

***Definition 3*** *The* Leadership group *is a subset of* $\mathcal{N}$. *Node* $j \in \mathcal{N}$ *belongs to the* Leadership group *if, for some* $i \in \mathcal{N}$ *it holds that* $(PR_i)_j \geq (PR_i)_k$ *for all* $k \neq j$. *i.e. for some personalization vector* $\mathbf{v}_i$ *node* $j$ *has the greatest PageRank.*

## 3. Modelling Social Competences

To fix ideas, let us consider the network shown in Figure 1. We assume that this network[1] represents a small SNS.

[1] Available at http://www.infovis-wiki.net/index.php/Social_Network_Generation

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| basic PR | 12 | 13 | 6 | 4 | 7 | 8 | 11 | 10 | 5 | 14 | 3 | 9 | 1 | 2 |
| ranking from (1) $\epsilon = 0.3$ | 4 | 12 | 6 | 13 | 8 | 7 | 11 | 10 | 1 | 2 | 9 | 3 | 5 | 14 |
| ranking from (2) $\epsilon = 0.8$ | 6 | 12 | 13 | 7 | 11 | 4 | 8 | 10 | 2 | 5 | 14 | 1 | 9 | 3 |

Table 1: Node ranking, in descending order, for the network of Figure 1 obtained when computing the indicated PageRank, using $\eta_i$ from (1) and (2).

Our idea consists in defining a personalization vector that is a linear combination of some prescribed social skills or social competences. In this model, a *social competence* is any feature of the user related to social skills that can enhance its PageRank as a node of an SNS.

Let $\eta_i \in [1, \infty[$ a quantity that measures the *social competence* of node $i$. The final goal of the present model is to give an explicit expression of the personalization vector in the form $\mathbf{v} = [\eta_1, \eta_2, \ldots, \eta_n]$, such that

$$\eta_i = f(\text{social skills of node i})$$

where $f$ is a linear function, which must be specified. In the computations we use the normalized version $\mathbf{v}/||\mathbf{v}||_1$.

In this network, taking $\epsilon = 0.3$ a computation shows that we have one Competitivity group, and one Leadership group. This means that by using the personalization vector we can enhance the PageRank of a user such that any user may be the leader. We can say we have a very democratic network: any node can win!

For this network, when $\epsilon = 0.3$ if we take

$$\eta_i = 6.3, \quad \text{if} \quad i = 4, \quad \text{and} \quad \eta_i = 1, \quad \text{else} \tag{1}$$

we obtain that node 4 is the winner; i.e. it has the biggest PageRank. See Table 1.

When $\epsilon = 0.8$ the network shows four Competitivity groups and the Leadership group is $\{6, 12, 13\}$. Let us image that node 6 has very high social skills and we want to enhance its ranking. We can use

$$\eta_i = 3.1, \quad \text{if} \quad i = 6, \quad \text{and} \quad \eta_i = 1, \quad \text{else} \tag{2}$$

and then we obtain that node 6 is the winner, see Table 1.

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## References

[1] D. Lazer, *et al*, *Science* **323**, 721 (2009).

[2] J. Kleinberg, *Communications of the ACM* **51**, 66 (2008).

[3] R. Albert and A.L. Barabási, *Rev. Mod. Phys.* **74**, 47, (2002).

[4] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez and D.-U. Hwang, *Physics Reports* **424**, 175 (2006).

[5] C. T. Butts, *Science* **325**, 414 (2009).

[6] M. S. Granovetter, *The American Journal of Sociology* **78**, 1360 (1973).

[7] F. Pedroche, *Mathematical and Computer Modelling*, **52**, 1052, (2010).

[8] A. N. Langville and C. D. Meyer, *Google's Pagerank and Beyond: The Science of Search Engine Rankings. Princeton University Press*, (2006).