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Unsupervised shape classification by using a complex network-based methodology

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Abstract. This work introduces a complex network-based methodology for the unsupervised shape classification problem [5][3]. First, each image is modelled as a undirected graph, and then some descriptors (i.e. measurements) both from the complex networks theory [4] (in particular, degree and joint degree) and from image shape analysis (circularity) are extracted from the image graph. Next, some different similarity measures between pair of images are defined to create a (dense) complex network model that represents the similarities between the patterns in the dataset. After that, we produce a multi-scale network by the application of an edge pruning threshold. For each of these scales, we obtain a small-world network where the communities found represent the resulting clusters (i.e. the classes) of objects. The accuracy of the proposed shape classification methodology, that is robust to geometrical transformations on the images, was tested on the Brown University Shape Database achieving correct classification results above 96%.

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1. Introduction

The classification of objects is an important task in Image Analysis[5]. In many cases, the regions containing the objects are first segmented and then some features like shape, texture or colour are extracted. The shape attributes are essential for the case of classifying binary images (i.e. black and white ones). Shape of planar objects can be mainly described using their contours or using skeletons[3]. When handling the contour information to classify objects, there can exist a large linear and non-linear variabilities among the patterns of the same class, that make the shape classification a challenging problem. Shape classification can be carried out in a supervised form (i.e. where the classes are predetermined) or in an unsupervised form (i.e. where the classes are not known in advance and similarities between patterns are searched during the classification in order to determine clusters of patterns).

This paper presents a new application of Complex Network theory [2] to the unsupervised shape classification problem. Given a database of binary images, the contour of each image is described as an undirected graph [6]. The pattern characterization is performed by extracting three types of histogram features [1] from its graph: node degree, joint degree and circularity histograms, respectively. After that, each pair of images in the dataset is compared using the corresponding histograms for each of the three features considered to obtain three confusion matrices (i.e. one matrix for each feature). These matrices that represent the distance between pairs of patterns (i.e. shapes) are then combined into one unique matrix using different linear and non-linear combination schemes. Using a varying threshold, we determine the clusters of patterns produced, and compare them to the ground truth data to determine the classification accuracy results. The proposed solution present the advantage to be a general methodology for unsupervised shape classification that produces accurate results when the involved algorithm components (i.e. selected features from shapes and distance-matrices fusion schemes) are selected appropriately.

2. Proposed shape classification methodology

To model an image I as a network graph $G(I) = (V, E)$, each pixel is initially mapped to a vertex of V. Given two graph nodes i and j, corresponding to the respective image pixels p_i and p_j, these pixels are connected if their distance is smaller or equal than a radius r value. The connection weight w for the considered edge is defined as follows:

$$w(p_i, p_j) = \begin{cases} |I(p_i) - I(p_j)| & \text{if } dist(p_i, p_j) \leq r \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

These intensity differences between connected pairs of pixels are computed for all arcs at a distance smaller or equal than the radius. As the considered shape images are binary, the only possible connections are defined when the difference of intensity between neighbour pixels is one, which means that the connected adjacent nodes have different intensity values.

Once the image graph is built for each pattern image, the respective graph histograms are computed for the three features considered in this work. The node degree and joint degree correspond to features of the Complex Network theory. The *degree* (or *connectivity*) d_i of a node i is the number of edges directly connected to it. The *jointdegree* of a pair of nodes i and j, with respective degrees d_i and d_j, is the probability $P(d_i, d_j)$ of existing an edge connecting both nodes. The third feature used is an image processing one called *circularity* which represents the percentage of object pixels contained in concentric circles centred in the shape's centre-of-mass. Figure 1 shows one shape pattern corresponding to the *camel* class and the corresponding histogram produced for the *jointdegree* feature.

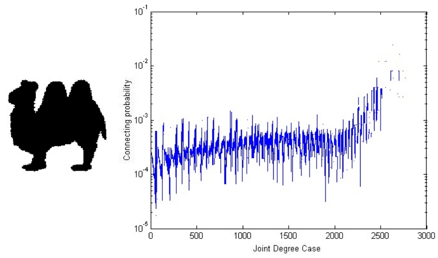

Figure 1: A *camel* shape and its corresponding joint degree feature histogram.

Given two object shapes i and j, and a feature l, the respective histograms of the objects for this feature are $hist_i^l$ and $hist_j^l$. The distance between these shape histograms $dist^l(i,j)$ for feature l is computed using the following formula:

$$dist^l(i,j) = \frac{\sum_{k=0}^{N} \left| hist_i^l(k) - hist_j^l(k) \right|}{\sum_{k=0}^{N} max(hist_i^l(k), hist_j^l(k))} \tag{2}$$

where: k represents the number of $N+1$ histogram bins.

After calculating the distances $dist^l(i,j)$ between all pairs of objects in the dataset (of size M) for each feature l (where: $l = 1..3$), we obtain the three distance matrices M_l. These matrices can be combined into a unique one M using different possible fusion schemes. In this work, the following schemes were tested: maxima, average, squared average, several weighted sums and the Fisher score combination. We considered six possible weighted sums: the first three ones consisted of giving 80% of weight to one feature and 10% to the two other ones, and the three remaining ones consisted of giving 60% of weight to one feature and 20% to the two other ones. The Fisher combination method give weights to the features according their Fisher ratio (i.e. the quotient between the inter-class variance and the intra-class variance), and the sum of weights is one.

For each of the possible distance matrices M, computed according to the fusion schemes explained, we calculated different filtered matrices M_{th} according to the values of filtering thresholds th. Each value of the matrix M is compared to the considered threshold th (where: $0 \leq th \leq 1$) and set to 0 if this value is greater than the threshold. Finally, the resulting classification matrices M_{th} are compared to the ground truth matrix G to determine the accuracy of each classification method.

The proposed Complex Networks-based method for unsupervised classification is summarized by the following algorithm.

3. Experiments

Our experiments were performed on the Shape Indexing of Image Database [7] developed at Brown University (USA). This shape dataset contains 17 classes of binary images with 12 images per class. We analyzed separately the classification performance of each individual feature in unsupervised shape classification (i.e. degree, joint degree and circularity, respectively). Next, the combination of features using the explained fusion schemes were also tested. For all the cases, the best recognition results were achieved using a threshold value in the interval: $0.2 \leq th \leq 0.4$. When using only one individual feature for unsupervised classification, the best results were achieved by the $jointdegree$ feature with produced 5.24% of miss-classifications for a threshold of $th = 0.3$. When combining the three features for classification purposes, the combination giving 60% of weight to circularity and 20% to both node degree and joint degree features produced only a 3.01% classification errors for a threshold value of $th = 0.2$.

Table 1 presents the confusion matrix results corresponding to the sixth weighted combination tested (i.e. this fusion scheme gives 80% of weight to the joint degree feature and 10% to both node degree and circularity features) for a threshold value of $th = 0.3$.

Algorithm 1: The algorithm proposed

Input :

- Database (I_i) of shape images (containing $L = M \times N$ images, with $M = 17$ classes and $N = 12$ images per class
- Ground truth matrix of classes G_{LL}

begin
  for $i = 1$ **to** L **do**
    Model image contour I_i as a complex network graph CN_i;
    Extract shape and complex-network descriptors D_{ik} for each $k = 1, \cdots, 3$ and each image I_i (i.e. node degree, joint degree and circularity, respectively);
  end
  for $k = 1$ **to** 3 **do**
    compute three distance matrices (one for each descriptor used);
    for $i = 1$ **to** L **do**
      for $j = 1$ **to** L **do**
        Compute the distance matrix C_{kij} between images i and j for each descriptor k by using the corresponding histograms $H(D_{ik})$ and $H(D_{jk})$;
      end
    end
  end
  Combine the distance descriptor matrices C_k with $1 \leq k \leq 3$ into a unique distance matrix M by using a fusion function f (such as max, average, squared average, weighted convex combination or Fisher ratio);
  for $th = 0$ **to** 1 **step** 0.05 **do**
    Compute the filtered distance matrix M_{th} by removing some links from M beyond th;
    Analyze the classification errors by comparing M_{th} with the ground truth matrix of classes G, producing confusion matrices with the classification results.;
  end
end

Output: Correct classification results (%) for each given fusion method f and threshold th considered

	c 1	c 2	c 3	c 4	c 5	c 6	c 7	c 8	c 9	c 10	c 11	c 12	c 13	c 14	c 15	c 16	c 17
c 1	11	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
c 2	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c 3	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c 4	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0
c 5	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0
c 6	0	0	0	0	0	7	0	1	0	0	0	2	0	0	2	0	0
c 7	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0
c 8	1	0	0	0	0	1	0	9	0	0	0	1	0	0	0	0	0
c 9	0	0	0	0	0	0	0	0	11	0	0	1	0	0	0	0	0
c 10	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0
c 11	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0
c 12	0	0	0	0	0	1	0	1	0	0	0	8	0	0	2	0	0
c 13	0	0	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0
c 14	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	0	0
c 15	0	0	0	0	0	1	0	0	0	0	0	3	0	0	8	0	0
c 16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0
c 17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12

Table 1: Confusion matrix for the weighted combination giving 80% to the joint degree and 10% to the other features.

4. Conclusions

We presented an novel application of the complex network theory to the unsupervised shape classification problem. Our approach can be seen as a general methodology for the considered problem. It can be adapted to other datasets by considering additional features (i.e. complex network-based and/or image-based ones) and other possible graph fusion schemes. The implemented solution has been successfully tested on a difficult dataset of shape images producing high accurate classification results (i.e. above 96%). One future work will consist in adapting the proposed pattern classification methodology to the multiplex framework.

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