

Evolution of brain network structure under a critical condition as induced by local currents

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Abstract. The brain may be viewed as a complex network which, at the functional level, is highly heterogeneous and disassortative, with scale-free distributed degrees [1]. This is likely to be determined, at least in part, by physiological processes inducing “synaptic pruning”. Here, we propose a new model of evolving neural systems in which synaptic pruning is mediated by microscopic rules that depend on physiological information associated to the dynamics of both neurons and synapses. A preliminary study shows that only when the system reaches a memory phase there is a correlation between each neuron degree and the synaptic current it is receiving. Furthermore, the final state of the evolving network shows, in this memory regime, critical features such as a scale-free node degree distribution.

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1. Introduction

Synaptic pruning is a physiological process that takes place in the brain of many mammals during development, and which consists on an extensive activity-dependent elimination of synapses [1, 2, 3]. Several advantages of this kind of network-building process have been proposed. First of all, reducing the number of synapses implies a drastic reduction in energy consumption [3].

Secondly, this strategy substantially reduces the amount of genetic information required for coding in the neural network, since instead of instructing for precise connections, more general rules can be applied.

Synaptic pruning has been observed in many brain areas of different species, and it is expected to play a mayor role in brain development [2]. Experimental studies have shown that the resulting network after pruning shows properties of scale-invariance, heterogeneity and dissasortativity [1], a structure which is known to improve both the robustness and efficiency of a network [3] .

In this work, we seek for a deep correlation between neural network structure and brain activity patterns, in accordance with previous studies [4]. In particular, we will study the correlation between the topological properties and the cognitive abilities of a neural network which evolves under a process of synaptic pruning.

2. Model definition

Consider a simple undirected network with N nodes defined by the adjacency matrix e_{ij} . Each node can be characterized by its degree, $k_i = \sum_{j=1}^N e_{ij}$. Initially, the degrees follow some given distribution $p(k, t = 0)$ with mean p_0 .

The activity of neurons in the network will be modeled by the Hopfield model [5], which is simple enough to provide a fast computation whereas presenting measurable cognitive properties, such as associative memory. Each neuron will be characterized by its state $s_i = \{-1, 1\}$ (silent or firing respectively). Within this model, the synapses are characterized by their synaptic weight, defined by the Hebb rule:

$$w_{ij} = \frac{1}{\langle k \rangle} \sum_{\mu=1}^P \xi_i^\mu \xi_j^\mu \quad (1)$$

when there are P patterns of network activity (or memories) ξ_i^μ to be learned, and $\langle k \rangle$ is the mean degree of the network. The local field felt by each neuron is therefore

$$h_i = \sum_{j=1}^N w_{ij} e_{ij} s_j \quad (2)$$

and the model assumes a probabilistic dynamics for neuron states as follows:

$$P(s_i = \pm 1) = \frac{1}{2} [1 \pm \tanh(\beta(h_i - \theta_i))] \quad (3)$$

where $\theta_i = \frac{1}{2} \sum_{j=1}^N w_{ij}$ is the neuron threshold for firing and β is the noise parameter controlling neuron excitability.

The evolution of the network is defined by the probability each node has to gain or loose an edge, P_i^g and P_i^l , which will be considered to factorize in two terms. The first one is a global term which characterizes the total number of synapses in the network. This quantity is known to be determined by the concentrations of different molecules which can diffuse through large areas of tissue, and will be therefore considered global. The second term is a local term which will take into account that each neuron's gain and loose probabilities should depend also on its activity [6].

The next assumption of the model will be the dependence and explicit form of these probabilities. In order to couple the topological and physiological dynamics, they should be functions of the network activity, what has been observed experimentally. Since neuron activity is determined by its local field, h_i , and network energy intake can be also considered a function of the synaptic current, we will assume these probabilities to be:

$$P_i^g = u(I_m) \pi(I_i) \quad (4)$$

$$P_i^l = d(I_m) \sigma(I_i) \quad (5)$$

where we have defined the synaptic current arriving at a neuron to be

$$I_i = |h_i - \theta_i| \quad (6)$$

and the mean current, $I_m = \frac{1}{N} \sum_{i=1}^N I_i$.

The global probabilities will be chosen so that they can reproduce the exponential decay to equilibrium found in experimental data [6]:

$$u(I_m) = \frac{n}{N} \frac{I_m(t)}{2I_E} \quad d(I_m) = \frac{n}{N} \left(1 - \frac{I_m(t)}{2I_E} \right) \quad (7)$$

where n and I_E are two parameters that define the speed of the pruning and the mean value of the stationary field (and connectivity).

The local probabilities will determine the resulting stationary topology. For the sake of simplicity, power-laws will be considered, since they captured the essence of a wide class of non-linear monotonous functions.

The macroscopic state of the system will be characterized by the overlap of the network state with the memorized patterns, m^μ , as well as with the distribution of degrees $p(k)$.

3. Results and conclusions

Preliminary simulations of our system depict a rich emergent phenomenology depending on the level of stochasticity in the network and the number of

memorized patterns. Among other effects, three different characteristic phases have been found, which correspond to very well known phases of the standard Hopfield model. Interestingly in our model, the existence of these phases implies differences of the resulting steady-state network topology as well as of the network activity.

The first phase occurs in the low noise regime and for few memorized patterns. It is a "memory phase" in the sense that the stationary activity state of the network resembles one of the memorized patterns. Once one or these memories is recalled, the system evolves to create a direct correlation between each node's degree and local field (figure 1) associated to the neuron placed at that node. This in fact suggests a strong correlation between network structure and its neural activity. Furthermore, this correlation allows us to find a set of local probabilities for which the network evolves towards a heterogeneous "critical" state with a scale-free distribution of degrees, such as those found in nature.

The second phase appears with high noise level, and it consists on a non-memory phase in which network evolution is governed by noise, which leads to a stationary homogeneous "subcritical" distribution of degrees. Finally, the last phase appears when the number of memorized is increased but the noise is kept low. It is a spin-glass phase in which the stationary activity state of the network is not one of the memorized patterns, but a combination of some of them. This leads to a mixture between the previous dynamics which could lead to a "supercritical" distribution of degrees, with the majority of nodes having low degrees (storing low memories information) but some of them presenting very high degree values (storing high memory information). Even more, network evolution can destabilize the equilibrium in this phase and get the system to vague between different states (data not shown).

In conclusion, the model proposed here relates in a natural way structural topologies and functionality of neural systems. Also it can lead to a scale-free distribution of degrees such as those found in nature if there exists a direct correlation between neurons degree and the local field or synaptic current these neurons are receiving, which is provided by the associative memory property. A deepest analysis of the system is currently under development, and it will include an exhaustive analysis of the phase space of the system.

References

- [1] V. EGUÍLUZ, D. CHIALVO, G. A. CECCHI, M. BALIKI, AND A. APKARIAN, *Physical Review Letters*. **94**, 018102 (2005).
- [2] N. GOGTAY ET AL., *Proc. Natl. Acad. Sci U.S.A.* **101**, 8174-8179 (2004).

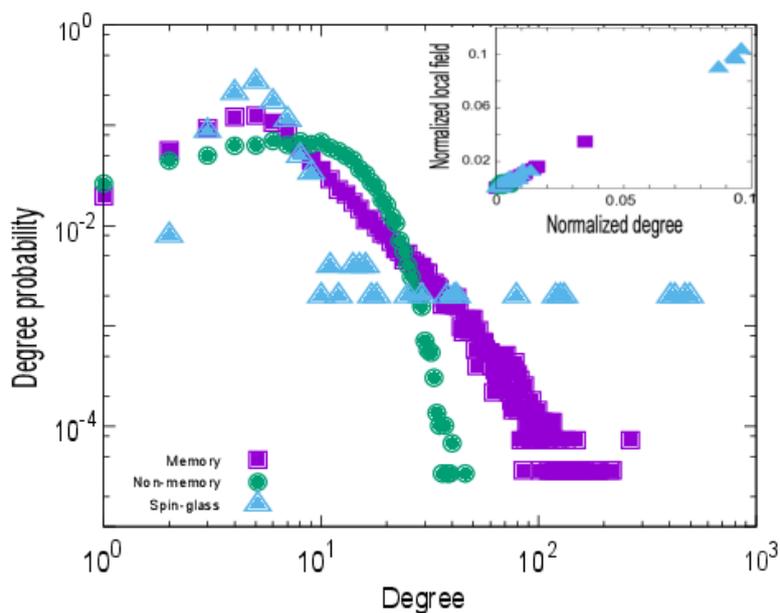


Figure 1: Steady-state degree distributions found for the memory (purple square data points), non-memory (green circle data points) and spin-glass (blue triangle data points) phases described in the text. In the inset, the local field of each neuron is plotted as a function of its degree for these three phases. This shows that in the subcritical case only the low degree nodes store memory information (there are not nodes with high degree). In the supercritical phase, there are nodes with small degree which store low memory information and nodes with high degree that store high memory information. In the critical phase the system is heterogeneous so there are nodes with any degree storing any level of memory information.

- [3] S. NABLAHA, A. L. BARTH, Z. BAR-JOSEPH, *PLOS Computational Biology* **11**(7), 1004347 (2015).
- [4] J.J. TORRES AND J. MARRO, *Scientific Reports* **5**, 12216 (2015).
- [5] J.J. HOPFIELD, *Proc. Natl. Acad. Sci. U. S. A.* **79**, 2554 (1982).
- [6] S. JOHNSON, J.J. TORRES AND J. MARRO, *Journal of Statistical Mechanics* **10**, 03003 (2010).